	A population is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{P}{10}\left(1 - \frac{P}{15}\right)$. a) If P(0) = 3, what is the $\lim_{t \to \infty} P(t)$? If P(0) = 20, what is the $\lim_{t \to \infty} P(t)$? b) If P(0) = 3, for what value of P is the population growing the fastest? Rogawski 8. If y(x) is a solution to $\frac{dy}{dx} = 3y(10 - y)$ with y(0) = 3 then as $x \to \infty$, A) y(x) increases to ∞ B) y(x) increases to 5 C) y(x) decreases to 5 D) y(x) increases to 10 E) y(x) decreases to 10 9. If y(x) is a solution to $\frac{dy}{dx} = 4y(12 - y)$ with y(0) = 10 then as $x \to \infty$, A) y(x) decreases to ∞ B) y(x) increases to 6 C) y(x) increases to 12 D) y(x) decreases to 12 E) y(x) decreases to 0 16. If $\frac{dy}{dt} = 3y(10 - 2y)$ with y(0) = 1 then, y is increasing the fastest when A) y = 1.5 B) y = 2.5 $6y(5-y)$ C) y = 3 D) y = 4 E) y = 5 18. If $\frac{dy}{dt} = 3y(10 - 2y)$ with y(0) = 1, then the maximum value of y is A) y = 1 B) y = 2.5 capacity ~~C)~~ y = 5 D) y = 10 E) Never attained; has no maximum value

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25. Given the differential equation $\frac{dz}{dt} = z\left(6 - \frac{z}{50}\right)$, where z(0) = 50, what is the $\lim_{t\to\infty} z(t)$?

A) 50 B) 100 C) 300 D) 6 E) 200

25. Given the differential equation $\boxed{\frac{dz}{dt} = z\left(6 - \frac{z}{50}\right)}$, where z(0) = 50, then z is increasing the fastest when z =

A) 150 B) 100 C) 300 D) 50 E) 100

Other Rate type problems

Rogawski

11. The rate at which a certain disease spreads is proportional to the quotient of the percentage of the population with the disease and the percentage of the population that does not have the disease. If the constant of proportionality is .03, and y is the percent of people with the disease, then which of the following equations gives R(t), the rate at which the disease is spreading.

A) R(t) = .03y

B) $R(t) = \frac{.03dy}{dt}$

C) $\frac{dr}{dt} = \frac{.03R}{(1-R)}$

D) $R(t) = .03\frac{y}{(1 - y)}$

Have disease

Do not have

E) $\frac{dr}{dt} = .03R$

multiply

12. The rate of change of the volume, V, of water in a tank with respect to time, t is directly proportional to the square root of the time, t, it takes to empty the tank . Which of the following is a differential equation that describes this relationship.

~~A)~~ $V(t)=k\sqrt{t}$ ~~B)~~ $V(t)=k\sqrt{V}$ C) $\frac{dV}{dt}=k\sqrt{t}$

~~D)~~ $\frac{dV}{dt}=\frac{k}{\sqrt{V}}$ ~~E)~~ $\frac{dV}{dt}=k\sqrt{V}$

16. Let P(t) represent the number of wolves in a population at time t years, when $t \geq 0$. The population P(t) is increasing at rate directly proportional to 500 divided by P(t), where the constant of proportionality is k. Write the differential equation that describes this relationship.

$$\frac{dP}{dt}=k\cdot\frac{500}{P(t)}$$

23. If P(t) is the size of a population at time t, which of the following differential equations describes exponential growth in the size of the population.

Linear A) $\frac{dP}{dt}=200$ B) $\frac{dP}{dt}=200t$ quad C) $\frac{dP}{dt}=100t^2$ cubic

D) $\frac{dP}{dt}=200P$ E) $\frac{dP}{dt}=100P^2$

a) $\frac{dP}{dt}=200$

$\int dP=\int 200\,dt$

$P=200t+C$

b) $\frac{dP}{dt}=200t$

$\int dP=\int 200t\,dt$

$P=100t^2+C$

D) $\frac{dP}{dt}=200P$

$\int \frac{dP}{P}=\int 200\,dt$

$\ln P=200t+C$

$\boxed{P=e^{200t+C}}$

$\frac{dP}{dt}=100P^2$

$\int \frac{dP}{P^2}=\int 100\,dt$

$\int P^{-2}\,dP=\int 100\,dt$

$-P^{-1}=100t+C$

$-\frac{1}{P}=100t+C$

$\boxed{\frac{-1}{100t+C}=P}$